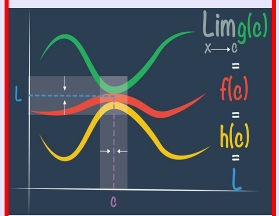
Math 261

Spring 2023

Lecture 31



Feb 19-8:47 AM

Given $f(x) = \frac{1}{x}$ $\quad x \neq 0 \to$ Vertical Asymptote $\to x = 0$

Domain $(-\infty, 0) \cup (0, \infty)$

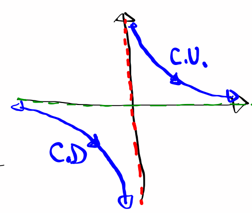
x-Int $\to$ None , y-Int $\to$ None

$\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -\infty} f(x) = 0 \Rightarrow$ Horizontal Asymptote $\to y = 0$

$f'(x) = \frac{-1}{x^2}$ $\qquad f''(x) = \frac{2}{x^3}$

$f'(x)$ is undefined at $x = 0$ $\qquad f''(x)$ is undefined at $x = 0$

x	$-\infty$	0	∞
$f'(x)$	$-$	O	$-$
$f''(x)$	$-$	O	$+$
$f(x)$			



$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 0^-} f(x) = -\infty$

Apr 11-8:46 AM

$$f(x) = \frac{x}{x^2+4}$$

$x^2+4 \geq 4 \to x^2+4 \neq 0 \to$ Domain $\to (-\infty, \infty)$

x-Int. $\to y=0 \to f(x)=0 \to \dfrac{x}{x^2+4}=0 \to x=0$ $\to (0,0)$

Y-Int $\to x=0 \to f(0) = \dfrac{0}{0^2+4} = 0 \longrightarrow (0,0)$

$f(x)$ is an odd function $f(-x) = \dfrac{-x}{(-x)^2+4} = \dfrac{-x}{x^2+4} = -f(x)$

$f(x)$ is symmetric w/r the origin.

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x}{x^2+4} = 0$, $\lim\limits_{x \to -\infty} f(x) = 0 \Rightarrow$ H.A. $y=0$

Apr 11-8:56 AM

$$f(x) = \frac{x}{x^2+4}$$

$$f'(x) = \frac{1(x^2+4) - x \cdot 2x}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} \checkmark$$

$(2+x)(2-x)$

$+$

$f'(x)=0 \to 4-x^2=0 \to x = \pm 2$

$f'(x)$ is never undefined $\to x^2+4 \neq 0$

$$f''(x) = \frac{-2x(x^2+4)^2 - (4-x^2)\cdot 2(x^2+4)^1 \cdot 2x}{\left[(x^2+4)^2\right]^2}$$

$$= \frac{-2x\cancel{(x^2+4)}\left[x^2+4+2(4-x^2)\right]}{(x^2+4)^{\cancel{4}\,3}} = \frac{-2x(12-x^2)}{(x^2+4)^3}$$

$x^2+4+8-2x^2 = 12-x^2$

$$= \frac{2x(x^2-12)}{(x^2+4)^3}$$

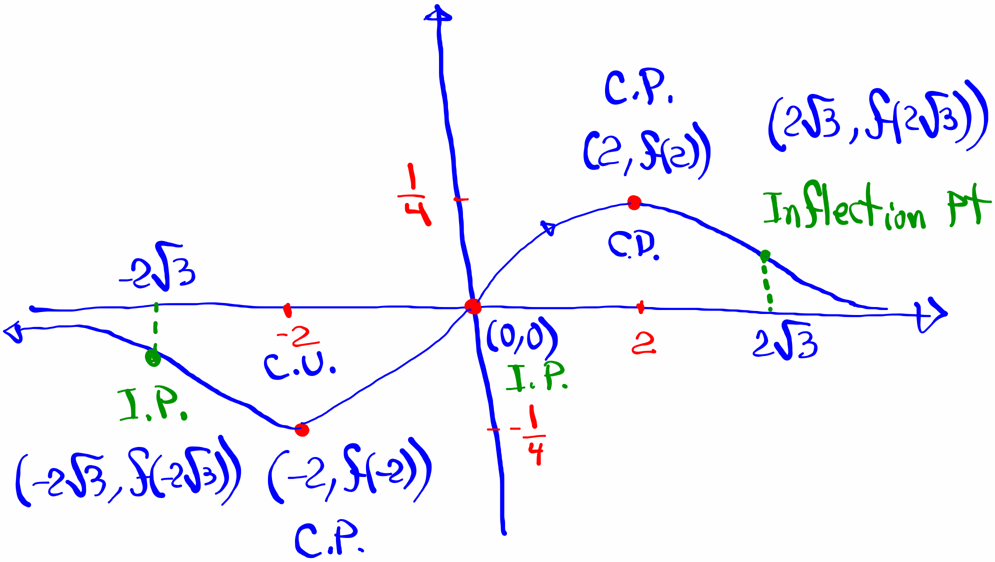
$f''(x)=0 \to x=0,\ x=\pm\sqrt{12} = \pm 2\sqrt{3}$

$f''(x)$ is never undefined $\to x^2+4 \neq 0$

x	$-\infty$ to $-2\sqrt{3}$	$-2\sqrt{3}$ to -2	-2 to 0	0 to 2	2 to $2\sqrt{3}$	$2\sqrt{3}$ to ∞
$f'(x)$	−	−	+	+	−	−
$f''(x)$	−	+	+	−	−	+
$f(x)$	↘	↘	↗	↗	↘	↘

I.P. (at $-2\sqrt{3}$) I.P. (at 0) I.P. (at $2\sqrt{3}$)

Apr 11-9:02 AM



Apr 11-9:17 AM

Find two numbers whose difference is 10, and the product is minimum.

$x - y = 10 \rightarrow y = x - 10$ $\quad y = 5 - 10$ $\quad \boxed{y = -5}$

Product xy is minimum

$xy = x(x - 10) \leftarrow$ Minimum

C.U.

$f(x) = x(x-10) = x^2 - 10x$

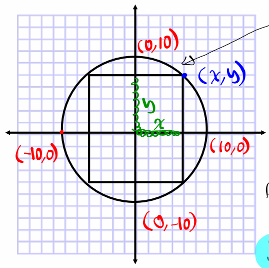
$f'(x) = 2x - 10 \rightarrow f'(x) = 0 \rightarrow \boxed{x = 5} \rightarrow$ Min.

$f''(x) = 2 > 0 \rightarrow$ C.U.

$\{5 \text{ \& } -5\}$

Apr 11-9:40 AM

Consider a Circle with radius 10.



$x^2 + y^2 = 100 \; \begin{cases} \to y^2 = 100 - x^2 \\ \to y = \sqrt{100-x^2} \end{cases}$

This rectangle to have max. area.

Area = $\boxed{4xy}$

Area = $4x \cdot \sqrt{100-x^2}$

$f(x) = 4x\,(100-x^2)^{1/2}$

$$f'(x) = 4\left[1\cdot(100-x^2)^{1/2} + x \cdot \cancel{\tfrac{1}{2}}(100-x^2)^{-1/2}\cdot(\cancel{2}x)\right]$$

$$= 4\left[\sqrt{100-x^2} - \frac{x^2}{\sqrt{100-x^2}}\right] = 4\left[\frac{\sqrt{100-x^2}\cdot\sqrt{100-x^2}}{\sqrt{100-x^2}} - \frac{x^2}{\sqrt{100-x^2}}\right]$$

$$= \frac{4\left[100-x^2-x^2\right]}{\sqrt{100-x^2}} = \frac{4\overbrace{\left[100-2x^2\right]}^{-2(x^2-50)}}{\sqrt{100-x^2}} = \frac{-8(x^2-50)}{\sqrt{100-x^2}}$$

$f'(x) = 0 \to x = \pm\sqrt{50}$

$-$ $\quad -\sqrt{50}$ $\quad +$ $\quad \sqrt{50}$ $\quad -$

Min. Max

Max. at $x = \sqrt{50}$

$x^2 + y^2 = 100$

$(\sqrt{50})^2 + y^2 = 100 \to y = \sqrt{50}$

Max. Area

$4xy$

$4\sqrt{50}\sqrt{50} = \boxed{200}$